Securities Insight for Attorneys
Monte-Carlo Analysis: A Tool for Evaluating Investment Returns

By Steve Pomerantz

Monte-Carlo is a statistical technique that is very useful in its application to a wide variety of problems, all of which involve a sense of uncertainty in their outcome. This form of analysis allows us to identify probabilities that are associated with those outcomes that may be of interest. It is important to recognize that certain transactions and analyses can not even be understood outside of the context of Monte-Carlo. Even for those situations that do not explicitly require this type of analysis, Monte-Carlo contributes enormously to one's understanding and ultimately to the decision-making process. Our primary interest here is in applying this type of analysis to situations involving investment decisions.

In particular, options and other derivatives are complicated financial transactions, each contract with its own nuance in terms of how they will affect an investments' performance or even whether or not the transaction is entered into on fair and reasonable terms.

For example, recent litigation surrounding certain options and derivative-related transactions within tax shelters involves a discussion on whether or not particular investments possess a "reasonable chance of earning a reasonable profit," as required by tax codes. While Monte-Carlo does not offer a definition of what reasonable profit means it allows one to quantify the likelihood of profit as well as the magnitude of those profits.

As another example, consider strategies that have been used to manage concentrated stock positions. How should one evaluate the relative merits of holding a large stock position, partially selling some of the stock or engaging in some type of hedging strategy be it a cost-less collar or a pre-paid forward transaction?

In addition to offering insight on investment potential, Monte-Carlo analysis provides an alternative picture of investment risk by providing different information than the standard notion of investment risk, or standard deviation. Options can be used to hedge investments but they can also be used to create additional leverage within an investment structure. Monte-Carlo is a very effective tool in identifying if a portfolio or transaction contains more or less risk than initially thought.

While standard deviation is certainly the most popular measure of risk, there is plenty we can learn about an investment by using other methods to examine possible returns. In this article we illustrate how Monte-Carlo can be used to gain insight on the qualitative behavior of an investment by identifying some of the non-traditional measures of investment performance.

While measures like expected return and volatility are very common, they offer only limited insight on investment possibilities.

As a simple example, consider the following. Suppose we have three different investments each held over a 3-month period with three equally likely outcomes for the three scenarios listed in Table 1 below.

Table 1

	Investment A	Investment B	Investment C
Scenario 1	24%	28%	14%
Scenario 2	0%	-14%	14%
Scenario 3	-24%	-14%	-28%

The traditional measures of investment return and risk will provide only limited insight. Each of these investments has an expected return of 0%, and a risk as measured by the standard deviation of returns of 20%. Yet measured by other objective investment measures we can see a different picture as Table 2 illustrates:

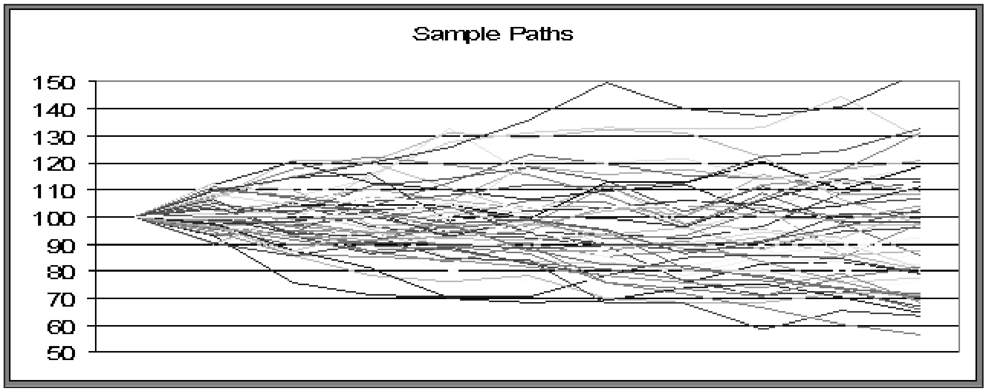
Table 2

	Investment A	Investment B	Investment C
Probability of Positive Return	1/3	2/3	1/3
Probability of Zero Return	1/3	0	0
Probability of Negative Return	1/3	1/3	2/3

For an investor that is averse to negative returns, Investment C is the most risky, in fact twice as risky as the other two possibilities, though traditional risk measures would not identify this.

Monte-Carlo analysis provides a very effective way to picture the range of possible investment outcomes. This technique is also very useful because it allows one to analyze not just option transactions, but much more complicated transactions as well. The general techniques can be applied to a wide array of security transactions.

Table 3



For example, the same analysis that is applied to a simple stock investment can be applied to very complicated "exotic" option investments, such as those embedded in recent tax shelter products.

The basic idea of Monte-Carlo is to randomly sample stock prices through the holding period of an investment and observe exactly what cash flows occur. As we choose more and more sample paths, a distribution begins to appear illustrating the range and frequency of possible outcomes for the investment. In this manner, we can create a probability distribution of outcomes rather than just providing descriptive statistics, such as expected return or standard deviation.

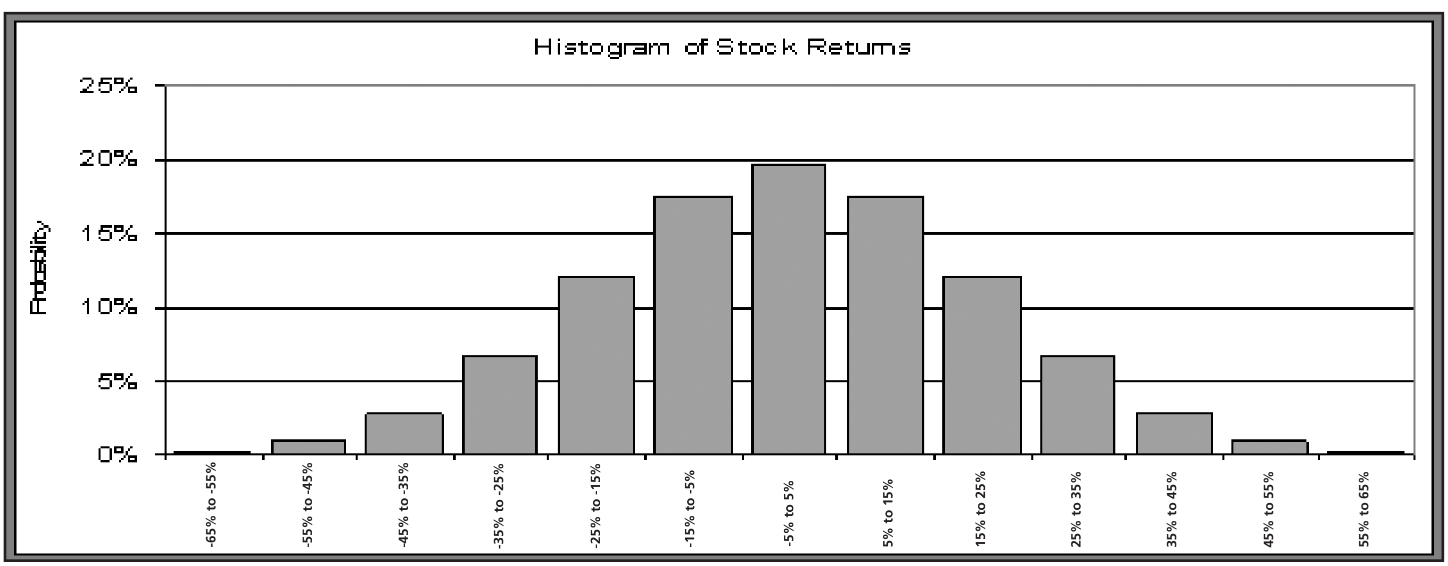
The important part of this technique is in selecting the paths in the simulation. In other words, what constitutes an appropriate mechanism of selecting the paths from which to determine the distribution of likely results? The standard approach in securities pricing is to use the assumptions that are embedded in the Black-Scholes methodology which assumes that stock returns follow a normal distribution, with an expected return equal to the prevailing risk-free (treasury) rate and a standard deviation that can be calculated from the movements of the stock or stocks underlying the subject investments. While other applications of Monte-Carlo will rely on different types of probability distributions and parameters, the choices stated above are rather standard throughout the financial industry.

Table 3 illustrates what a representative set of paths would look like using this approach. Each path follows a random path for some specified period of time, all starting at the same point. The dispersion of terminal prices is governed by the choice of standard deviation that is used in the Monte-Carlo process.

On the final date, the stock prices form the familiar picture shown below in Table 4. Extreme returns, both positive and negative, are less likely, while returns closer to zero are the most likely outcomes.

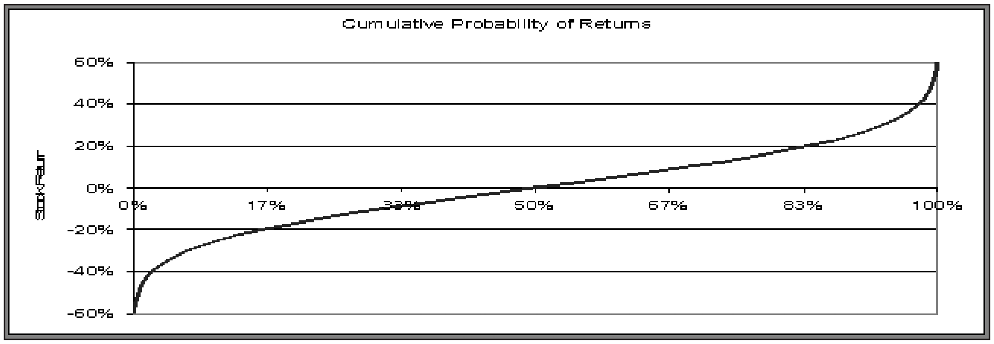
In addition to displaying the data as a histogram of return possibilities, we can also illustrate the cumulative distribution of returns for the simple investment of purchasing a single stock. This is similar to Investment A mentioned previously, because the investment returns are symmetric.

Table 4



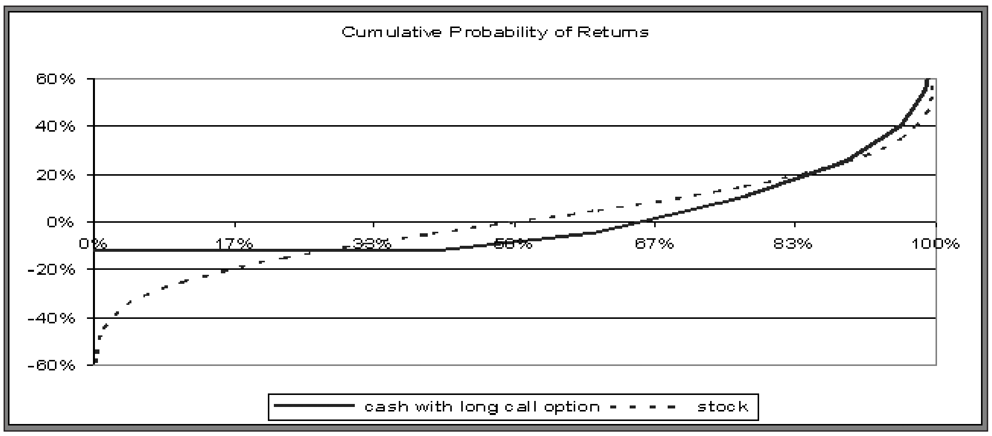
Several features of this investment can be read from the graph in Table 5. The 50th percentile, or median, investment return is 0%, and extreme returns are -60% and 60%. Additionally, the inner two-thirds of the investment, measured from the 17th to 83rd percentile, is between -20% and 20%. These values are a result of the assumptions that were used in generating the paths for the simulation.

Table 5



We can generate similar analyses for the most complicated of investment strategies as well, but for now let's consider a simple option strategy. Rather than investing $100 in a stock which has the outcome illustrated above, suppose one were to purchase a certain amount of at-the-money call options on that stock, with a three-month maturity. Assume that $14 was invested in those options and that the remaining $86 was held in cash. The cumulative return as illustrated in Table 6 is compared with the stock investment as well. In this manner, we can view the investment profile of both investments at the same time and visualize their respective properties. We can see how often the option strategy under performs the stock investment and by how much. We can visualize the limited downside of the options strategy and how it eventually outperforms a direct stock investment for large enough movements of the stock. This is typical of an investment like Investment B, where the strategy will lose money if the stock declines or remains unchanged, but can have significant upside if the stock appreciates.

Table 6



As the graph illustrates, there is a 67% chance that the return will be less than zero, in other words that the investor will not receive back the original investment. But there is also a 20% chance that the stock will decline and that a simple stock purchase will under perform the downside protected call strategy.

As another example, suppose we consider a strategy where the call option is sold in exchange for a premium of $14. If the stock declines or remains unchanged, then the premium is kept for an enhanced return. If the stock appreciates by too much then the option will go in-the-money and there will be a payment required of the investor, which may be quite large relative to the premium received. This is analogous to Investment C and is represented in Table 7. This graph illustrates that the probability that the investment return is negative is only 17%, yet the upside is significantly limited.

Table 7

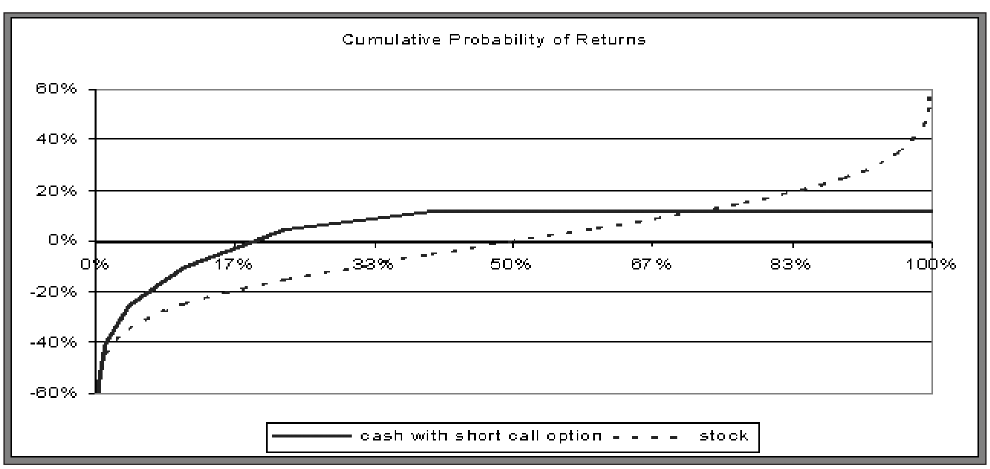
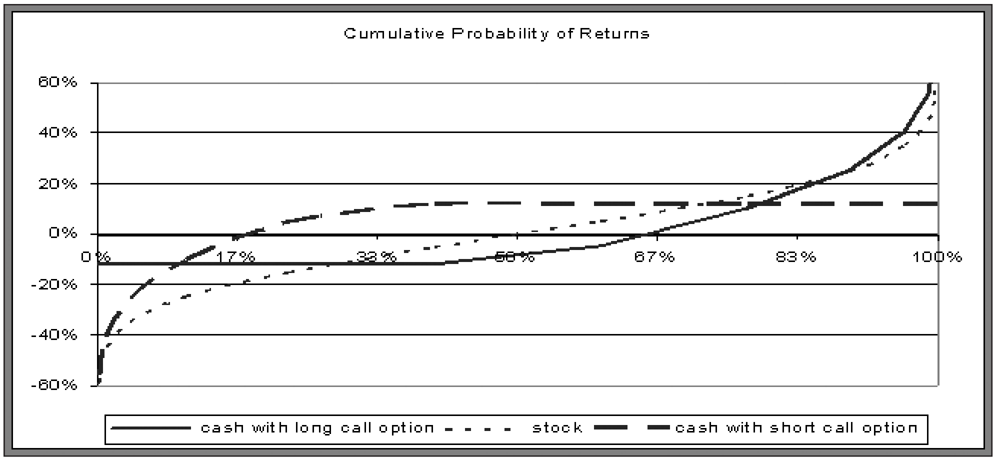
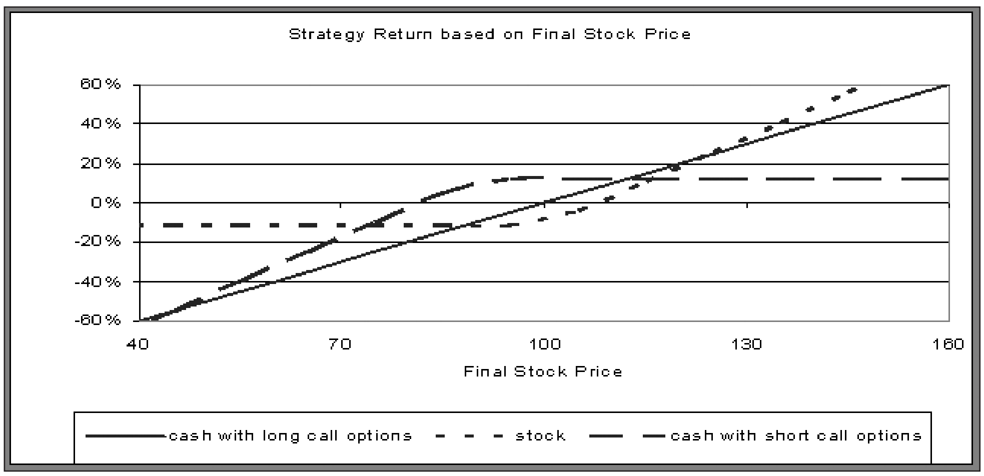


Table 8



And finally, Table 8 illustrates each of these strategies simultaneously to highlight how different they are. It is important to recognize, however, that each of these strategies has an expected return of 0% and a standard deviation, or risk, of 20%, yet clearly there are differences in both their quantitative and qualitative behaviors.

Table 9



By way of comparison Table 9 illustrates the traditional payoff diagrams for these three strategies. While the shape of each curve is similar, the use of Monte-Carlo provides not just a sense of possible returns but also the likelihood of each outcome.

The traditional types of diagrams like those above provide directional information as to how investments perform if the stock goes up or down. Monte-Carlo does not provide this analysis directly and so serves best to complement these more traditional methods of analysis.

What is presented with Monte-Carlo analysis is a complementary understanding of an investment's likely range of returns as well as extreme possibilities. Within litigation, this type of analysis is useful in providing as complete a picture as possible for both the quantitative and qualitative risks and opportunities embedded in an investment.

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